Answer the following questions. **CALCULATORS ARE NOT ALLOWED** Each question carries 10 marks.

1 a) Find $f'(x)$ if $f(x) = \sin^2\left(\sqrt{\dfrac{1+x}{1-x}}\right)$

b) Let

$$f(x) = \begin{cases} (x^2)\sqrt{1 + \dfrac{1}{x^2}} & \text{, if } x \neq 0 \\ \\ 0 & \text{, if } x = 0 \end{cases}$$

Use the definition of the derivative to determine whether $f'(0)$ exists or not.

2. a) Let

$$f(x) = \begin{cases} \dfrac{\cos x + 2x - 1}{x} & , x < 0 \\ \\ Ax + B & , 0 \leq x \leq 1 \\ \\ \dfrac{2(x^2 - 1)}{|x - 1|} & , x > 1 \end{cases}$$

Find the values of A and B so that f is continuous for all real numbers.

b) Show that $y = (x - 1)^{\frac{2}{3}} + 2$ has a cusp.

3 a) Find the equations of the tangent lines to the graph of $x\sin y + \cos x + y^2 = 5$ when $x = 0$

b) Use differentials to approximate $(-26.88)^{\frac{2}{3}}$.

a) Evaluate $\displaystyle\lim_{x \to 0} \dfrac{\tan^3 2x}{x^4 + 2x^3}$, if it exists.

b) A rocket is rising vertically at the speed of 2 km/min (see the figure). Find the rate of change of the angle θ when the rocket is 8 km above the ground.